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At the horizon of a supersymmetric AdS_5 black hole: Isometries and half-BPS giants

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Abstract: The near-horizon geometry of an asymptotically AdS_5 supersymmetric black hole discovered by Gutowski and Reall is analysed. After lifting the solution to 10 dimensions, we explicitly solve the Killing spinor equations in both AdS_2 Poincaré and global coordinates. It is found that exactly four supersymmetries are preserved which is twice the number for the full black hole. The full set of isometries is constructed and the isometry supergroup is shown to be $\mathrm{SU}(1,1|1)\times\mathrm{SU}(2)\times\mathrm{U}(3)$. We further study half-BPS configurations of D3-branes in the near-horizon geometry in Poincaré and global coordinates. Both giant graviton probes and dual giant graviton probes are found.

Keywords: Black Holes in String Theory, AdS-CFT Correspondence.

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1. Introduction

Asymptotically AdS_5, rotating, electrically charged supersymmetric black holes of minimal $D = 5$ gauged supergravity with regular horizons were first constructed by Gutowski and Reall in [1, 2]. These solutions have been further generalised in [3–6]. When lifted to 10-dimensional solutions of type IIB, these geometries asymptote to the maximally supersymmetric $AdS_5 \times S^5$ solution and preserve just two of the 32 supersymmetries [7]. One of the important outstanding problems in string theory is to account for the entropy of these black holes both from the string theory and the holographic boundary gauge theory points of view.

The standard way of counting the microstates of a supersymmetric black hole in string theory is to count the BPS states of the D-brane system in the asymptotic geometry of the black hole. In recent times it has been realised that the entropy of extremal black holes depends just on the string theory in the near horizon geometry called the attractor geometry of the black hole. Therefore, the Bekenstein-Hawking-Wald entropy should also be related to a certain number of appropriate BPS states in the attractor geometry or those in the holographically dual quantum mechanics. This program has been demonstrated successfully in the context of 4-dimensional extremal black holes in [8, 9] (see also [10] and [11]). For the supersymmetric AdS_5 black holes of [1, 2] , one expects two complementary approaches to count the microstates as well: (1) count the BPS states with the right charges in the $AdS_5 \times S^5$ background, (2) count an appropriate set of BPS states in the near-horizon geometry. The problem of counting BPS states with just two supersymmetries in $AdS_5 \times S^5$ geometry is a hard problem (see [12 − 14] where the problem of counting the BPS states with four supersymmetries was addressed, and [15] where a fermi-surface model was proposed to achieve qualitative agreement with the counting). In this paper, we initiate addressing the problem using the second approach.

We consider the single-parameter black holes with equal angular momenta [1] in AdS_5 directions and a single U(1) electric charge. When the angular momentum vanishes, this solution reduces to AdS_5. We lift the near-horizon geometry of the black hole to a solution of type IIB supergravity in ten dimensions. By studying the integrability condition of the Killing spinor equations, it is found that the number of supersymmetries of the near-horizon geometry is four, which is twice the number of supersymmetries of the full black hole. We explicitly construct the Killing spinor in both Poincaré and global coordinates. Using the Killing spinor solution and the technique of [16, 17], we show that the superisometry group is $\mathrm{SU}(1,1|1)$. We demonstrate that the full isometry supergroup of the 10-dimensional near-horizon solution is $\mathrm{SU}(1,1|1) \times \mathrm{SU}(2) \times \mathrm{U}(3)$. As one expects for an extremal black hole there is an AdS_2 factor in the near-horizon geometry and we consider both Poincaré and global coordinates for it.

We then initiate the study of probe branes in the near-horizon geometry along the lines of [11] in the context of BMPV back holes [18]. Two sets of probe D3 branes are found which preserve half of the near-horizon supersymmetries. These are the analogues of giant gravitons [19] and dual giant gravitons [20, 21] in $AdS_5 \times S^5$. The probes in Poincaré coordinates are static and have vanishing Hamiltonians. They still carry non-zero angular momenta because of the rotation of the background. The probes in global coordinates rotate and have non-zero angular momenta and Hamiltonians.

The paper is organised as follows. In section 2, we lift the near horizon solution to ten dimensions. In section 3, we solve the Killing spinor equation explicitly in Poincaré coordinates. In section 4, we derive the isometry supergroup of the geometry. In section 5, we consider the problem from the point of view of global coordinates and solve the Killing spinor equation in these coordinates. In section 6, we initiate the study of probe branes in Poincaré coordinates while in section 7, the probe branes are studied in global coordinates. We conclude with a brief discussion in section 8.

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2. The black hole and its near-horizon geometry

The metric of the five-dimensional solution with equal angular momenta is specified by the fünfbein [1]

$$
\begin{aligned}
e^0 &= \mathcal{F}\big(dt + \Psi\sigma_3^L\big)\,, \qquad\qquad e^1 = \mathcal{F}^{-1}\left(1+\frac{2\omega^2}{l^2}+\frac{r^2}{l^2}\right)^{-\frac{1}{2}} dr\,,\\
e^2 &= \frac{r}{2}\sigma_1^L\,, \quad e^3 = \frac{r}{2}\sigma_2^L\,, \quad e^4 = \frac{r}{2l}\sqrt{l^2+2\omega^2+r^2}\sigma_3^L\,.
\end{aligned} \tag{2.1}
$$

The right-invariant one-forms on SU(2) are $\sigma_1^L = \sin\phi\, d\theta - \sin\theta\,\cos\phi\, d\psi$, $\sigma_2^L = \cos\phi\, d\theta + \sin\theta\,\sin\phi\, d\psi$ and $\sigma_3^L = d\phi + \cos\theta\, d\psi$, where $0\le\theta\le\pi, 0\le\psi\le 2\pi, 0\le\phi\le 4\pi$. They satisfy $d\sigma_i^L = -\frac{1}{2}\epsilon_{ijk}\sigma_j^L\wedge\sigma_k^L$ with $\epsilon_{123}=1$. Furthermore

$$
\mathcal{F} = 1-\frac{\omega^2}{r^2}\,, \qquad \Psi = -\frac{\eta r^2}{2l}\left(1+\frac{2\omega^2}{r^2}+\frac{3\omega^4}{2r^2(r^2-\omega^2)}\right), \tag{2.2}
$$

with $\eta=\pm1$ and ω is constant. The 1-form gauge potential is given by

$$
A = \frac{\sqrt{3}}{2}\left[\mathcal{F}dt + \frac{\eta\omega^4}{4lr^2}\sigma_3^L\right]. \tag{2.3}
$$

We choose $\eta=1$ from here on. This solution asymptotes to global AdS_5 and in this limit reads

$$
e^0 = dt - \frac{r^2}{2l}\sigma_3^L, \quad e^1 = \frac{dr}{\sqrt{1+\frac{r^2}{l^2}}}, \quad e^2 = \frac{r}{2}\sigma_1^L, \quad e^3 = \frac{r}{2}\sigma_2^L, \quad e^4 = \frac{r}{2}\sqrt{1+\frac{r^2}{l^2}}\;\sigma_3^L \tag{2.4}
$$

with $F=dA=0$. This can be put into the standard form by writing $\tilde{\phi} = \phi + \frac{2t}{l}$ and $\tilde{t}=t$ which imply $\frac{\partial}{\partial t} = \frac{\partial}{\partial\tilde{t}} + \frac{2}{l}\frac{\partial}{\partial\tilde{\phi}}$. The black hole solution carries an electric charge under the U(1) gauge field given by

$$
Q = \frac{1}{4\pi G}\int_{S^3_\infty} \star F = \frac{\sqrt{3}\pi\omega^2}{2G}\left(1+\frac{\omega^2}{2l^2}\right). \tag{2.5}
$$

where G is the 5-dimensional Newton's constant. The black hole carries an angular momentum given by

$$
J = \frac{3\pi\omega^4}{8lG}\left(1+\frac{2\omega^2}{3l^2}\right), \tag{2.6}
$$

while the entropy is

$$
S_{\rm BH} = \frac{\pi^2}{2G}\omega^3\sqrt{1+\frac{3\omega^2}{4l^2}}\,, \tag{2.7}
$$

which may be written as [24]

$$
S_{\rm BH} = \pi\sqrt{l^2Q^2 - \frac{2\pi l^3}{G}\,|J|} = \pi\sqrt{l^2Q^2 - 4N^2\,|J|}\,, \tag{2.8}
$$

in terms of the electric charge and angular momentum of the black hole. Here $N^2 = \frac{\pi l^3}{2G}$. The near-horizon limit of this geometry is

$$\begin{aligned}
e^0 &= \frac{2r}{\omega} dt - \frac{3\omega^2}{4l}\sigma_3^L, \qquad e^1 = \frac{\omega l}{2\lambda}\frac{dr}{r}, \qquad e^2 = \frac{\omega}{2}\sigma_1^L, \quad e^3 = \frac{\omega}{2}\sigma_2^L, \\
e^4 &= \frac{\omega}{2l}\lambda\, \sigma_3^L, \qquad\qquad A = \frac{\sqrt{3}}{2}\left(\frac{2r}{\omega}dt + \frac{\omega^2}{4l}\sigma_3^L\right) = \frac{\sqrt{3}}{2}\left(e^0 + \frac{2\omega}{\lambda}e^4\right).
\end{aligned} \tag{2.9}$$

Here we have defined

$$\lambda = \sqrt{l^2 + 3\omega^2}\,. \tag{2.10}$$

The gauge field strength, $F = dA$ is given by

$$\begin{aligned}
F &= \frac{\sqrt{3}}{2l}\left[3e^{14} - e^{23} - \frac{2}{\omega}\lambda\, e^{01}\right], \\
\star F &= \frac{\sqrt{3}}{2l}\left[3e^{023} - e^{014} + \frac{2}{\omega}\lambda\, e^{234}\right].
\end{aligned} \tag{2.11}$$

The equations of motion are

$$\begin{aligned}
R_{ab} - 2F_{ac}F_b{}^c + \frac{1}{3}\left(F_{cd}F^{cd} + \frac{12}{l^2}\right)\eta_{ab} &= 0, \\
d\star F + \frac{2}{\sqrt{3}}F\wedge F &= 0.
\end{aligned} \tag{2.12}$$

where our convention for the Hodge dual is $\epsilon_{01234} = 1$. We will now lift this to a ten-dimensional solution. The lift formula is [25] (see also [26])

$$\begin{aligned}
ds_{10}^2 &= ds_5^2 + l^2\sum_{i=1}^{3}\left[(d\mu_i)^2 + \mu_i^2\left(d\xi_i + \frac{2}{l\sqrt{3}}A\right)^2\right], \\
F^{(5)} &= (1 + *_{(10)})\left[-\frac{4}{l}\mathrm{vol}_{(5)} + \frac{l^2}{\sqrt{3}}\sum_{i=1}^{3} d(\mu_i^2)\wedge d\xi_i \wedge *_{(5)}F^{(2)}\right],
\end{aligned} \tag{2.13}$$

where $\mu_1 = \sin\alpha$, $\mu_2 = \cos\alpha\,\sin\beta$, $\mu_3 = \cos\alpha\,\cos\beta$ with $0 \le \alpha \le \pi/2$, $0 \le \beta \le \pi/2$, $0 \le \xi_i \le 2\pi$ and together they parametrise S^5. Note that we define the Hodge star of a p-form ω in n-dimensions as $*_{(n)}\omega_{i_1\ldots i_{n-p}} = \frac{1}{p!}\epsilon_{i_1\ldots i_{n-p}}{}^{j_1\ldots j_p}\omega_{j_1\ldots j_p}$, with $\epsilon_{0123456789} = 1$ and $\epsilon_{01234} = 1$ in an orthonormal frame. The ten-dimensional geometry is specified by (2.9) together with

$$\begin{aligned}
e^5 &= l\, d\alpha, \qquad\qquad e^6 = l\,\cos\alpha\, d\beta, \\
e^7 &= l\,\sin\alpha\,\cos\alpha\,[d\xi_1 - \sin^2\beta\, d\xi_2 - \cos^2\beta\, d\xi_3], \\
e^8 &= l\,\cos\alpha\,\sin\beta\,\cos\beta\,[\,d\xi_2 - d\xi_3], \\
e^9 &= -\frac{2}{\sqrt{3}}A - l\,\sin^2\alpha\, d\xi_1 - l\,\cos^2\alpha\,(\sin^2\beta\, d\xi_2 + \cos^2\beta\, d\xi_3).
\end{aligned} \tag{2.14}$$

and the five form [25, 26, 7]

$$
\begin{aligned}
F^{(5)} &= -4l^{-1}[e^0\wedge e^1\wedge e^2\wedge e^3\wedge e^4 + e^5\wedge e^6\wedge e^7\wedge e^8\wedge e^9] \\
&\quad +\frac{2}{\sqrt{3}}(e^5\wedge e^7 + e^6\wedge e^8)\wedge(*_{(5)}F^{(2)} - e^9\wedge F^{(2)}) \\
&= -\frac{4}{l}(e^{01234}+e^{56789}) - \frac{1}{l}(e^{57}+e^{68})\wedge\Bigg[-3e^{023}+e^{014}-\frac{2}{\omega}\lambda\, e^{234} \\
&\qquad\qquad +e^9\wedge\left(3e^{14}-e^{23}-\frac{2}{\omega}\lambda\, e^{01}\right)\Bigg]. \qquad (2.15)
\end{aligned}
$$

3. The Killing spinor

In this section, we will solve the Killing spinor equation. The strategy will be to use the integrability condition to simplify the equations on a projected subspace. The ten-dimensional Killing spinor equation is [7]

$$
D_m\epsilon + \frac{i}{1920}\Gamma^{n_1n_2n_3n_4n_5}\Gamma_m F^{(5)}_{n_1n_2n_3n_4n_5}\epsilon = 0\,. \qquad (3.1)
$$

We record the useful identity

$$
\begin{aligned}
\frac{i}{1920}\Gamma^{n_1n_2n_3n_4n_5}F^{(5)}_{n_1n_2n_3n_4n_5}\Gamma_m &= \frac{i}{4l}\left[\Gamma_{01234}-\frac{1}{4}(\Gamma^{57}+\Gamma^{68})\left(3\Gamma_{023}-\Gamma_{014}-\frac{2\lambda}{\omega}\Gamma_{234}\right)\right] \\
&\quad \times(1+\Gamma_{11})\Gamma_m \\
&\equiv \frac{1}{2}\mathcal{M}(1+\Gamma_{11})\Gamma_a e^a_m\,. \qquad (3.2)
\end{aligned}
$$

Here m is a spacetime index while a is a tangent-space index. The integrability condition is [7]

$$
\begin{aligned}
&\left[R_{mns_1s_2} - \frac{1}{48}F^{(5)}{}_{ms_1r_1r_2r_3}F^{(5)}{}_{ns_2}{}^{r_1r_2r_3}\right]\Gamma^{s_1s_2}\epsilon \\
&+\left[\frac{i}{24}\nabla_{[m}F^{(5)}_{n]s_1s_2s_3s_4} + \frac{1}{96}F^{(5)}_{mnr_1r_2s_1}F^{(5)r_1r_2}{}_{s_2s_3s_4}\right]\Gamma^{s_1s_2s_3s_4}\epsilon = 0\,. \qquad (3.3)
\end{aligned}
$$

Using a computer algebra program it can be shown that these impose the constraints

$$
i\Gamma^{0149}\epsilon = \Gamma^{2357}\epsilon = \Gamma^{2368}\epsilon = -\Gamma^{5678}\epsilon = -\epsilon\,, \qquad (3.4)
$$

$$
\Gamma^{23}\epsilon = \Gamma^{57}\epsilon = \Gamma^{68}\epsilon = -i\epsilon\,. \qquad (3.5)
$$

Of these only three are independent which may be chosen to be

$$
\Gamma^{0149}\epsilon = i\epsilon\,, \quad \Gamma^{23}\epsilon = -i\epsilon\,, \quad \Gamma^{57}\epsilon = -i\epsilon\,. \qquad (3.6)
$$

From these projections it follows that the solution in (2.13), (2.15) preserves at most 4 supersymmetries of the possible 32. After some tedious but straightforward algebra, one

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can verify that on the constrained subspace the components of the Killing spinor equation simplify to:

$$\left(\partial_t - \frac{4i\lambda r}{\omega^2 l}\Gamma_4\Gamma_0 P_+\right)\epsilon = 0\,, \tag{3.7}$$

$$\left(\partial_r + \left[-\frac{3}{2\lambda}\frac{\omega}{2r}\Gamma_{04} + \frac{1}{2r}\Gamma_{09} - \frac{3}{2\lambda}\frac{\omega}{2r}\Gamma_{49}\right]\right)\epsilon = 0\,, \tag{3.8}$$

$$\partial_\phi\epsilon = 0\,, \quad \partial_\theta\epsilon = 0\,, \quad \partial_\psi\epsilon = 0\,, \tag{3.9}$$

$$\partial_\alpha\epsilon = 0\,, \quad \partial_\beta\epsilon = 0\,, \tag{3.10}$$

$$\left(\partial_{\xi_j} + \frac{i}{2}\right)\epsilon = 0\,, \quad \text{for} \quad j = 1,2,3\,, \tag{3.11}$$

where we define the projectors

$$P_\pm = \frac{1}{2}(1 \pm \Gamma_{09})\,,$$

so that $P_+P_- = 0 = P_-P_+$. All angular equations can be easily solved. This leads to the Killing spinor ansatz

$$\epsilon = e^{-\frac{i}{2}(\xi_1+\xi_2+\xi_3)}\epsilon(r,t)\,. \tag{3.12}$$

Then the solution to the t equation is

$$\epsilon(r,t) = e^{\frac{2i\lambda r t}{\omega^2 l}M_1}\epsilon_r(r)\,, \tag{3.13}$$

where $M_1 = -(\Gamma_{49} + \Gamma_{04}) = -\Gamma_{49}(1 + \Gamma_{09}) = -2\Gamma_{49}P_+$ and satisfies $M_1^2 = 0$. Plugging this into the r equation leads to

$$\partial_r\epsilon_r = \frac{3\omega}{2\lambda r}\Gamma_{49}P_+\epsilon_r - \frac{1}{2r}(P_+ - P_-)\epsilon_r\,. \tag{3.14}$$

Now writing $\epsilon_r = \epsilon_r^+ + \epsilon_r^-$ such that $\Gamma_{09}\epsilon^\pm = \pm\epsilon^\pm$ we get

$$\partial_r\epsilon_r^+ = -\frac{1}{2r}\epsilon_r^+\,, \tag{3.15}$$

$$\partial_r\epsilon_r^- = \frac{3\omega}{2\lambda r}\Gamma_{49}\epsilon_r^+ + \frac{1}{2r}\epsilon_r^-\,. \tag{3.16}$$

The first of these immediately leads to

$$\epsilon_r^+ = \frac{1}{\sqrt{r}}\epsilon_0^+\,, \tag{3.17}$$

where ϵ_0^+ is a constant positive chirality spinor. Plugging this into the second equation leads to

$$\epsilon_r^- = \sqrt{r}\epsilon_0^- - \frac{3\omega}{2\lambda\sqrt{r}}\Gamma_{49}\epsilon_0^+\,, \tag{3.18}$$

where ϵ_0^- is a constant negative chirality spinor. Thus the complete Killing spinor is

$$\epsilon = e^{-\frac{i}{2}(\xi_1+\xi_2+\xi_3)}\left(\sqrt{r}\epsilon_0^- + \frac{1}{\sqrt{r}}\left(1 - \frac{4i\lambda r t}{\omega^2 l}\Gamma_{49}P_+\right)\left(1 - \frac{3\omega}{2\lambda}\Gamma_{49}\right)\epsilon_0^+\right)\,. \tag{3.19}$$

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Here $\epsilon_0^{\pm}$ are subjected to the same projection conditions as ϵ. The novelty here compared to the full black hole is the appearance of the other chirality ϵ_0^{+} in the solution. Alternatively, this result can be expressed compactly as:

$$\epsilon = e^{-\frac{4i\lambda rt}{\omega^2 l}\Gamma_{49}P_+} e^{\left(\frac{3\omega}{2\lambda}\Gamma_{49}P_+ - \frac{1}{2}\Gamma_{09}\right)\ln r}\epsilon_0 \,. \tag{3.20}$$

It is sometimes useful to split the solution in terms of Γ_{09} chiralities:

$$\begin{aligned} \epsilon^+ &= e^{-\frac{i}{2}(\xi_1+\xi_2+\xi_3)}\frac{1}{\sqrt{r}}\epsilon_0^+ \,, \\ \epsilon^- &= e^{-\frac{i}{2}(\xi_1+\xi_2+\xi_3)}\left(\sqrt{r}\epsilon_0^- - \frac{1}{\sqrt{r}}\left(\frac{3\omega}{2\lambda} + \frac{4i\lambda rt}{\omega^2 l}\right)\Gamma_{49}\epsilon_0^+\right) \,. \end{aligned} \tag{3.21}$$

Thus we conclude that the 10-dimensional lift of the near-horizon geometry of the black hole under consideration preserves precisely four supersymmetries with the explicit Killing spinors in eqs. (3.21). We next turn to computing the isometry superalgebra of this geometry.

4. Isometry supergroup

In the present section we shall need the basis vectors dual to the ten-dimensional frame of the near-horizon geometry in Poincaré coordinates. These are:

$$\tilde{e_0} = \frac{\omega}{2r}\partial_t - \frac{1}{l}(\partial_{\xi_1} + \partial_{\xi_2} + \partial_{\xi_3}) \,, \quad \tilde{e_1} = \frac{2\lambda r}{l\omega}\partial_r \,, \tag{4.1}$$

$$\tilde{e_2} = \frac{1}{\omega}\left(2\sin\phi\partial_\theta + 2\cot\theta\cos\phi\partial_\phi - 2\cos\phi\,\mathrm{cosec}\;\theta\partial_\psi\right) \,, \tag{4.2}$$

$$\tilde{e_3} = \frac{1}{\omega}\left(2\cos\phi\partial_\theta - 2\cot\theta\sin\phi\partial_\phi + 2\sin\phi\,\mathrm{cosec}\;\theta\partial_\psi\right) \,, \tag{4.3}$$

$$\tilde{e_4} = \frac{3\omega^2}{4\lambda r}\partial_t + \frac{2l}{\omega\lambda}\partial_\phi - \frac{2\omega}{\lambda l}(\partial_{\xi_1} + \partial_{\xi_2} + \partial_{\xi_3}) \,, \quad \tilde{e_5} = \frac{1}{l}\partial_\alpha \,, \quad \tilde{e_6} = \frac{1}{l}\sec\alpha\partial_\beta \,, \tag{4.4}$$

$$\tilde{e_7} = \frac{1}{l}\left(\cot\alpha\partial_{\xi_1} - \tan\alpha\partial_{\xi_2} - \tan\alpha\partial_{\xi_3}\right) \,, \quad \tilde{e_8} = \frac{1}{l}\left(\cot\beta\sec\alpha\partial_{\xi_2} - \tan\beta\sec\alpha\partial_{\xi_3}\right) \tag{4.5}$$

$$\tilde{e_9} = -\frac{1}{l}(\partial_{\xi_1} + \partial_{\xi_2} + \partial_{\xi_3}) \,. \tag{4.6}$$

Following the prescription in [16, 17], we now turn to the computation of the Killing spinor bilinears $\bar{\epsilon}\Gamma^i\epsilon$ which are Killing vectors. For the ten-dimensional complex Weyl representation of definite Γ_{09}-chirality, one can show that

$$\bar{\epsilon}^{\pm}\Gamma^a\epsilon^{\pm} = 0 \,, \tag{4.7}$$

unless $a = 0$ or $a = 9$. Conversely

$$\bar{\epsilon}^{\mp}\Gamma^a\epsilon^{\pm} = 0 \,, \tag{4.8}$$

if $a = 0$ or $a = 9$. Define

$$c = -\frac{3\omega}{2\lambda} - \frac{4i\lambda rt}{\omega^2 l} \,. \tag{4.9}$$

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First consider $a = I$ where $I \neq 0$ or 9. We have

$$\bar{\epsilon}\Gamma^I\epsilon = \bar{\epsilon}_0^-\Gamma^I\epsilon_0^+ + \bar{\epsilon}_0^+\Gamma^I\epsilon_0^- + \frac{1}{r}\bar{\epsilon}_0^+(c\Gamma^I\Gamma_{49} - c^*\Gamma_{49}\Gamma^I)\epsilon_0^+ \,. \tag{4.10}$$

Next consider $a = z$ where $z = 0$ or 9. This gives

$$\bar{\epsilon}\Gamma^z\epsilon = r\bar{\epsilon}_0^-\Gamma^z\epsilon_0^- + \bar{\epsilon}_0^- c\Gamma^z\Gamma_{49}\epsilon_0^+ - \bar{\epsilon}_0^+ c^*\Gamma_{49}\Gamma^z\epsilon_0^- + \frac{1}{r}\bar{\epsilon}_0^+(\Gamma^z - cc^*\Gamma_{49}\Gamma^z\Gamma_{49})\epsilon_0^+ \,. \tag{4.11}$$

Thus we have,

$$\begin{aligned}(\bar{\epsilon}\Gamma^a\epsilon)\tilde{e}_a \;=\; & \bar{\epsilon}_0^-\Gamma^0\epsilon_0^- r(\tilde{e}_0 - \tilde{e}_9) + \bar{\epsilon}_0^-\Gamma_4\epsilon_0^+(c(\tilde{e}_0 - \tilde{e}_9) + \tilde{e}_4 + i\tilde{e}_1) + \bar{\epsilon}_0^-\Gamma^A\epsilon_0^+\tilde{e}_A \\ & + \bar{\epsilon}_0^+\Gamma_4\epsilon_0^-(c^*(\tilde{e}_0 - \tilde{e}_9) + \tilde{e}_4 - i\tilde{e}_1) + \bar{\epsilon}_0^+\Gamma^A\epsilon_0^-\tilde{e}_A \\ & + \bar{\epsilon}_0^+\Gamma^0\epsilon_0^+\frac{1}{r}(\tilde{e}_0 + \tilde{e}_9 + (c + c^*)\tilde{e}_4 - i(c - c^*)\tilde{e}_1 + cc^*(\tilde{e}_0 - \tilde{e}_9)) \\ & + \bar{\epsilon}_0^+\Gamma_{49}\Gamma^A\epsilon_0^+\frac{1}{r}(c - c^*)\tilde{e}_A \,,\end{aligned} \tag{4.12}$$

where A takes all values from 2 to 8 except 4. The terms involving A vanish as we show now. Note that the spinor $\epsilon_0^\pm$ can be written as $(1 + i\Gamma_{23})(1 + i\Gamma_{57})(1 + i\Gamma_{68})\epsilon_0/8$ so that we may always pull a suitable one of these projectors through Γ^A which then changes its chirality and annihilates the conjugate on the left. We have therefore shown that there are four independent coefficients of the form $\bar{\epsilon}_0^\pm\Gamma^a\epsilon_0^\pm$ contained in the Killing spinor bilinears. This demonstrates that there are four bosonic generators in the superisometry group. In other words, each of these generators corresponds to the coefficient of a certain linear combination of $z_1z_1^*$, $z_1z_{32}^*$, $z_1^*z_{32}$ and $z_{32}z_{32}^*$ where z_1, z_{32} are taken as the two independent complex components of the Killing spinor. Thus the independent Killing vectors are:

$$k^{(1)} = r(\tilde{e}_0 - \tilde{e}_9) \,, \tag{4.13}$$

$$k^{(2)} = c(\tilde{e}_0 - \tilde{e}_9) + \tilde{e}_4 + i\tilde{e}_1 \,, \tag{4.14}$$

$$k^{(3)} = c^*(\tilde{e}_0 - \tilde{e}_9) + \tilde{e}_4 - i\tilde{e}_1 \,, \tag{4.15}$$

$$k^{(4)} = \frac{1}{r}(\tilde{e}_0 + \tilde{e}_9 + (c + c^*)\tilde{e}_4 - i(c - c^*)\tilde{e}_1 + cc^*(\tilde{e}_0 - \tilde{e}_9)) \,. \tag{4.16}$$

These vectors can be easily verified to be Killing. Expressed in the holonomic frame these are:

$$k^{(1)} = \frac{\omega}{2}\partial_t \,, \tag{4.17}$$

$$k^{(2)} = \frac{-2i\lambda}{\omega l}(t\partial_t - r\partial_r) + \frac{2l}{\omega\lambda}\partial_\phi - \frac{2\omega}{\lambda l}\partial_{\underline{\xi}} \,, \tag{4.18}$$

$$k^{(3)} = \frac{2i\lambda}{\omega l}(t\partial_t - r\partial_r) + \frac{2l}{\omega\lambda}\partial_\phi - \frac{2\omega}{\lambda l}\partial_{\underline{\xi}} \,, \tag{4.19}$$

$$k^{(4)} = \frac{\omega(3l^2 + \lambda^2)}{8\lambda^2}\frac{1}{r^2}\partial_t + 8\frac{\lambda^2}{\omega^3 l^2}t^2\partial_t - \frac{16\lambda^2}{l^2\omega^3}rt\partial_r - \frac{6l}{\lambda^2}\frac{1}{r}\partial_\phi - \frac{2l}{\lambda^2}\frac{1}{r}\partial_{\underline{\xi}} \,. \tag{4.20}$$

All these Killing vectors are null. Rescaling $k^{(j)}$ by $\frac{2i\lambda}{\omega l}$ and defining $\frac{1}{2}(k^{(2)} - k^{(3)}) = \mathcal{J}$, $\frac{1}{2}(k^{(2)} + k^{(3)}) = Z$, $k^{(1)} = E^+$, $k^{(4)} = E^-$, we get the non-zero commutators

$$\left[\mathcal{J}, E^\pm\right] = \pm\, E^\pm \,,\; [\mathcal{J}, Z] = [Z, E^\pm] = 0 \,,\; \left[E^+, E^-\right] = 2\mathcal{J} \,. \tag{4.21}$$

This is the algebra $\mathfrak{sl}(2,R)\oplus\mathfrak{u}(1)$ where $E^\pm,\mathcal{J}$ are the generators of $\mathfrak{sl}(2,R)$ and Z is the generator of the $\mathfrak{u}(1)$ R-symmetry. Just as there is a bosonic charge $Q_B(k)$ associated with each isometry k of the solution, to each Killing spinor ϵ, there corresponds a fermionic charge $Q_F(\epsilon)$. The algebra of these is encoded in the decomposition of the bilinears constructed above in terms of the bosonic charges [16, 17] (see also [22, 23]). To extract the decomposition in a convenient form, we define the two linearly independent Killing spinors

$$\epsilon^{(1)} = \frac{1}{\sqrt{r}} e^{-\frac{i}{2}(\xi_1+\xi_2+\xi_3)}\left[\epsilon_0^+ + c\Gamma_{49}\epsilon_0^+\right] \tag{4.22}$$

$$\epsilon^{(2)} = \sqrt{r} e^{-\frac{i}{2}(\xi_1+\xi_2+\xi_3)}\epsilon_0^- \,. \tag{4.23}$$

We obtain the following linearly independent bilinears

$$\begin{aligned}
&\left(\bar\epsilon^{(2)}\Gamma^a\epsilon^{(1)}\right)\tilde e_a = (\bar\epsilon_0^-\Gamma^4\epsilon_0^+)\,k^{(2)}\,, \qquad &&\left(\bar\epsilon^{(2)}\Gamma^a\epsilon^{(2)}\right)\tilde e_a = (\bar\epsilon_0^-\Gamma^0\epsilon_0^-)\,k^{(1)}\\
&\left(\bar\epsilon^{(1)}\Gamma^a\epsilon^{(2)}\right)\tilde e_a = (\bar\epsilon_0^+\Gamma^4\epsilon_0^-)\,k^{(3)}\,, \qquad &&\left(\bar\epsilon^{(1)}\Gamma^a\epsilon^{(1)}\right)\tilde e_a = (\bar\epsilon_0^+\Gamma^0\epsilon_0^+)\,k^{(4)},.
\end{aligned} \tag{4.24}$$

Let us define the fermionic generators associated to the Killing spinors as follows

$$\epsilon^{(1)} \to Q^{(1)} \qquad \epsilon^{(2)} \to Q^{(2)}\,. \tag{4.25}$$

Then it immediately follows from (4.24) that

$$\left\{\bar Q^{(2)},Q^{(1)}\right\} = Z+\mathcal{J}\,,\ \left\{\bar Q^{(1)},Q^{(2)}\right\} = Z-\mathcal{J}\,,\ \left\{\bar Q^{(2)},Q^{(2)}\right\} = E^+\,,\ \left\{\bar Q^{(1)},Q^{(1)}\right\} = E^-\,. \tag{4.26}$$

All other odd-odd anti commutators are zero. These are in the standard $\mathfrak{sl}(2|1)$ form [27]. In addition to $k^{(1)},\dots,k^{(4)}$, there are also bosonic isometries of this solution which are not associated with the supergroup. To this end it can also be verified that the left-invariant vector fields (which generate right translations)

$$\begin{aligned}
\xi_1^R &= -\sin\psi\partial_\theta - \cot\theta\cos\psi\partial_\psi + \cos\psi\,\mathrm{cosec}\,\theta\partial_\phi\,,\\
\xi_2^R &= \cos\psi\partial_\theta - \cot\theta\sin\psi\partial_\psi + \sin\psi\,\mathrm{cosec}\,\theta\partial_\phi\,,\\
\xi_3^R &= \partial_\psi\,,
\end{aligned} \tag{4.27}$$

satisfying $[\xi_i^R,\xi_j^R] = -\epsilon_{ijk}\xi_k^R$ are Killing, reflecting the $\mathfrak{su}(2)_R$ isometries of the squashed S^3 of the near-horizon region. In addition to these we expect there to be more bosonic isometries coming from the S^5 part of the geometry that preserve the 1-form $i\sum_{k=1}^3 \bar z_i\,dz_i$ where $z_i = l\mu_i e^{i\xi_i}$ as before with $\mu_1=\sin\alpha$, $\mu_2=\cos\alpha\sin\beta$ and $\mu_3=\cos\alpha\cos\beta$. The following can be seen to be Killing vectors of our geometry.

$$\begin{aligned}
J_{13}+J_{24} &= -\cos\xi_{12}[\sin\beta\,\partial_\alpha - \tan\alpha\cos\beta\,\partial_\beta] + \sin\xi_{12}[\cot\alpha\sin\beta\,\partial_{\xi_1} + \tan\alpha\csc\beta\,\partial_{\xi_2}],\\
J_{14}-J_{23} &= \quad\ \sin\xi_{12}[\sin\beta\,\partial_\alpha - \tan\alpha\cos\beta\,\partial_\beta] + \cos\xi_{12}[\cot\alpha\sin\beta\,\partial_{\xi_1} + \tan\alpha\csc\beta\,\partial_{\xi_2}],\\
J_{15}+J_{26} &= -\cos\xi_{13}[\cos\beta\,\partial_\alpha + \tan\alpha\sin\beta\,\partial_\beta] + \sin\xi_{13}[\cot\alpha\cos\beta\,\partial_{\xi_1} + \tan\alpha\sec\beta\,\partial_{\xi_3}],\\
J_{16}-J_{25} &= \quad\ \sin\xi_{13}[\cos\beta\,\partial_\alpha + \tan\alpha\sin\beta\,\partial_\beta] + \cos\xi_{13}[\cot\alpha\cos\beta\,\partial_{\xi_1} + \tan\alpha\sec\beta\,\partial_{\xi_3}],\\
J_{35}+J_{46} &= -\cos\xi_{23}\,\partial_\beta + \sin\xi_{23}[\cot\beta\,\partial_{\xi_2} + \tan\beta\,\partial_{\xi_3}],\\
J_{36}-J_{45} &= \quad\ \sin\xi_{23}\,\partial_\beta + \cos\xi_{23}[\cot\beta\,\partial_{\xi_2} + \tan\beta\,\partial_{\xi_3}],\\
J_{12} &= \partial_{\xi_1},\qquad J_{34} = \partial_{\xi_2},\qquad J_{56} = \partial_{\xi_3}\,.
\end{aligned} \tag{4.28}$$

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where $\xi_{ij} = \xi_i - \xi_j$. These form the algebra $\mathfrak{u}(3)$. The algebra can be calculated using

$$[J_{ij},\, J_{mn}] = \delta_{in} J_{jm} + \delta_{jm} J_{in} - \delta_{im} J_{jn} - \delta_{jn} J_{im}. \tag{4.29}$$

We have checked that the Lie-derivative of the five form along all the above Killing vectors vanishes. Thus we have demonstrated that the isometry superalgebra of our near-horizon geometry is $\mathfrak{su}(1,1|1) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(3)$. Hence, we conclude that the isometry supergroup is $\mathrm{SU}(1,1|1) \times \mathrm{SU}(2) \times \mathrm{U}(3)$.

5. Global coordinates

We will now consider global coordinates. Let us first rewrite the five-dimensional part of the metric in Poincaré-like coordinates as follows:

$$\begin{aligned} ds^2 = &-\frac{4(1+\frac{3\omega^2}{l^2})}{\omega^2(1+\frac{3\omega^2}{4l^2})} r^2\, dt^2 + \frac{\omega^2}{4(1+\frac{3\omega^2}{l^2})}\frac{dr^2}{r^2} + \frac{\omega^2}{4}\Big((\sigma_1^L)^2 + (\sigma_2^L)^2\Big) \\ &+\frac{\omega^2}{4}\left(1+\frac{3\omega^2}{4l^2}\right)\left[\sigma_3^L + \frac{6}{\omega l\big(1+\frac{3\omega^2}{4l^2}\big)} r\, dt\right]^2 . \end{aligned} \tag{5.1}$$

We perform the coordinate transformation[1]

$$t \;=\; \frac{\sqrt{b^2+\rho^2}\;\; \sin\frac{\tau}{b}}{a[-\rho + \sqrt{b^2+\rho^2}\;\; \cos\frac{\tau}{b}]}, \qquad r = -\rho + \sqrt{b^2+\rho^2}\;\; \cos\frac{\tau}{b}\,, \tag{5.2}$$

$$\tilde{\phi} := \phi + \frac{6ab^3}{\omega l}\log\frac{b + \sqrt{b^2+\rho^2}\;\; \sin\frac{\tau}{b}}{b\cos\frac{\tau}{b} - \rho\sin\frac{\tau}{b}}\,. \tag{5.3}$$

Here $a^2 = \frac{4\lambda^2}{\omega^2 l^2 (1+\frac{3\omega^2}{4l^2})}$ and $b^2 = \frac{\omega^2 l^2}{4\lambda^2}$. This brings the metric into the form

$$ds^2 = -\left(1+\frac{\rho^2}{b^2}\right)d\tau^2 + \frac{d\rho^2}{1+\frac{\rho^2}{b^2}} + \frac{\omega^2}{4}\Big((\tilde{\sigma}_1^L)^2 + (\tilde{\sigma}_2^L)^2\Big) + \frac{\omega^2}{4}\left(1+\frac{3\omega^2}{4l^2}\right)\left(\tilde{\sigma}_3^L - \frac{6ab}{\omega l}\rho\, d\tau\right)^2, \tag{5.4}$$

where $\tilde{\sigma}_i^L$'s have $\tilde{\phi}$ in their definition. The AdS_2 part of the metric is now manifestly in global form. The gauge field reads

$$A = -\frac{\sqrt{3}}{2}\left[\frac{\omega^2}{4l}\tilde{\sigma}_3^L - \frac{2}{\omega ab}\rho\, d\tau\right], \tag{5.5}$$

after a gauge transformation. We choose the tangent space basis to be

$$e^0 = f\ d\tau, \quad e^1 = f^{-1} d\rho, \quad e^2 = \frac{\omega}{2}\tilde{\sigma}_1^L, \quad e^3 = \frac{\omega}{2}\tilde{\sigma}_2^L, \quad e^4 = \frac{\omega}{2ab}\left(\tilde{\sigma}_3^L - \frac{6ab}{\omega l}\rho\, d\tau\right). \tag{5.6}$$

where $f = \sqrt{1+\frac{\rho^2}{b^2}}$. For notational convenience we will drop the tilde from now. In this basis the field strength and its Hodge dual associated with A are

$$F = -\frac{\sqrt{3}}{2}\left[\frac{2}{\omega ab}\ e^{01} - \frac{1}{l}e^{23}\right], \quad \star F = \frac{\sqrt{3}}{2}\left[\frac{2}{\omega ab}\ e^{234} + \frac{1}{l}e^{014}\right]. \tag{5.7}$$

[1] To cover the full range of r, t the range of ρ and τ should be between $-\infty$ to ∞.

These satisfy the equation $d\star F + \frac{2}{\sqrt{3}} F\wedge F = 0$. After the 10-dimensional lift, the five-form reads

$$F^{(5)} = -\frac{4}{l}(e^{01234} + e^{56789}) + \frac{1}{l}\left(e^{57} + e^{68}\right) \wedge \left[e^{014} - e^{239} + \frac{2l}{\omega ab}\left(e^{234} + e^{019}\right)\right]. \tag{5.8}$$

The projection conditions following from integrability in global coordinates turn out to be:

$$\Gamma^{0149}\epsilon = -i\epsilon\,, \quad \Gamma^{2357}\epsilon = \Gamma^{2368}\epsilon = \Gamma^{5678}\epsilon = \epsilon\,, \tag{5.9}$$

$$\Gamma^{23}\epsilon = i\epsilon\,, \quad \Gamma^{57}\epsilon = \Gamma^{68}\epsilon = -i\epsilon\,. \tag{5.10}$$

showing, again, that at most four supersymmetries are preserved by the near-horizon geometry. Note that these conditions are almost the same as, but nevertheless different from, the corresponding ones in Poincaré coordinates. The flux contributes

$$\begin{aligned}\frac{i}{1920}\Gamma^{n_1n_2n_3n_4n_5}F^{(5)}_{n_1n_2n_3n_4n_5}\Gamma_m &= \frac{i}{4l}\left[\Gamma_{01234} + \frac{1}{4}(\Gamma^{57} + \Gamma^{68})\left(-\Gamma_{014} + \frac{2l}{\omega ab}\Gamma_{234}\right)\right] \\ &\quad \times(1+\Gamma_{11})\Gamma_m \\ &\equiv \frac{1}{2}\mathcal{M}_G(1+\Gamma_{11})\Gamma_a e^a_m \end{aligned} \tag{5.11}$$

to the Killing spinor equation. Here m is a spacetime index while a is a tangent-space index. Using these we get the following simplified component equations:

$$\left(\partial_\tau - \frac{i\rho}{2b^2}\Gamma_{49} + f\left(\frac{1}{\omega ab}\Gamma_{19} + \frac{3i}{2l}\Gamma_{09}\right)\right)\epsilon = 0\,, \tag{5.12}$$

$$\left(\partial_\rho + \frac{1}{lf}\hat{M}\right)\epsilon = 0\,, \tag{5.13}$$

$$\partial_\theta\epsilon = 0\,, \quad \partial_\phi\epsilon = 0\,, \quad \partial_\psi\epsilon = 0\,, \tag{5.14}$$

$$\partial_\alpha\epsilon = 0\,, \quad \partial_\beta\epsilon = 0\,, \tag{5.15}$$

$$\left(\partial_{\xi_j} + \frac{i}{2}\right)\epsilon = 0\,, \quad \text{for } \; j = 1,2,3\,. \tag{5.16}$$

where $\hat{M} = \frac{2b}{l}(\frac{3}{2}\Gamma_{04} + \frac{l}{\omega ab}\,\Gamma_{09})$, which satisfies $\hat{M}^2 = 1$. Again, all the angular equations are trivial and may be integrated immediately. Let us now solve the ρ equation to write

$$\epsilon(\tau,\rho) = e^{-\frac{1}{2}\sinh^{-1}\frac{\rho}{b}\,\hat{M}}\epsilon(\tau)\,, \tag{5.17}$$

where $\sinh^{-1}x = \log[x + \sqrt{1+x^2}]$. Then to solve the τ equation let us first rewrite the equation in the following form

$$\partial_\tau\epsilon = \frac{i}{2b}[\frac{\rho}{b}\Gamma_{49} - f\;\hat{M}\Gamma_{49}]\epsilon\,, \tag{5.18}$$

where we make use of the projection $\Gamma_{0149}\epsilon = i\epsilon$ to eliminate Γ_{19} in favour of Γ_{04}. Then it is straightforward to verify that the spinor

$$\epsilon(\tau,\rho) = e^{-\frac{1}{2}\sinh^{-1}\frac{\rho}{b}\,\hat{M}}e^{-\frac{i}{2}\hat{M}\Gamma_{49}\frac{\tau}{b}}\epsilon_0\,, \tag{5.19}$$

where ϵ_0 satisfying all the projections conditions is a solution to the Killing spinor equation. This solution can be split in terms of $\hat{M}\epsilon_0^{\pm} = \pm\epsilon_0^{\pm}$ as

$$\epsilon = \left(e^{-\frac{\chi}{2}}\cos\frac{\tau}{2b} + ie^{\frac{\chi}{2}}\sin\frac{\tau}{2b}\Gamma_{49}\right)\epsilon_0^{+} + \left(e^{\frac{\chi}{2}}\cos\frac{\tau}{2b} - ie^{-\frac{\chi}{2}}\sin\frac{\tau}{2b}\Gamma_{49}\right)\epsilon_0^{-}\,, \tag{5.20}$$

where $\rho = b\sinh\chi$.

Supergroup in global coordinates

The supergroup in global coordinates can be computed in the same manner as was done in the Poincaré coordinates. The basis vectors dual to the global vielbein are the same as in Poincaré coordinates with the exception of

$$\tilde{e_0} = \frac{1}{f}\partial_\tau + \frac{6ab}{l\omega f}\rho\partial_\phi - \frac{2ab}{\omega l f}\rho(\partial_{\xi_1} + \partial_{\xi_2} + \partial_{\xi_3})\,, \quad \tilde{e_1} = f\partial_\rho\,, \tag{5.21}$$

$$\tilde{e_4} = \frac{2ab}{\omega}\partial_\phi + \frac{\omega ab}{2l^2}(\partial_{\xi_1} + \partial_{\xi_2} + \partial_{\xi_3})\,. \tag{5.22}$$

In the same way as in section 4, one can use the constraints from the integrability condition to show that the only nonzero bilinears are $(\bar\epsilon\Gamma_0\epsilon)$,$(\bar\epsilon\Gamma_1\epsilon)$, $(\bar\epsilon\Gamma_4\epsilon)$ and $(\bar\epsilon\Gamma_9\epsilon)$. In addition we can use the condition $\hat{M}\epsilon_0^{\pm} = \pm\epsilon_0^{\pm}$ to derive the following relations

$$\begin{array}{ll} (\bar\epsilon_0^{\pm}\Gamma_9\epsilon_0^{\pm}) = \frac{2l}{3\omega ab}(\bar\epsilon_0^{\pm}\Gamma_4\epsilon_0^{\pm})\,, & (\bar\epsilon_0^{\pm}\Gamma_9\epsilon_0^{\mp}) = -\frac{3\omega ab}{2l}\,(\bar\epsilon_0^{\pm}\Gamma_4\epsilon_0^{\mp}) \\ (\bar\epsilon_0^{\pm}\Gamma_0\epsilon_0^{\pm}) = \mp\frac{l}{3b}(\bar\epsilon_0^{\pm}\Gamma_4\epsilon_0^{\pm})\,, & (\bar\epsilon_0^{\pm}\Gamma_1\epsilon_0^{\mp}) = \pm\frac{ia\omega}{2}(\bar\epsilon_0^{\pm}\Gamma_4\epsilon_0^{\mp})\,. \end{array} \tag{5.23}$$

With the aid of these we compute the independent bilinears.

$$\begin{aligned} &(\bar\epsilon\Gamma^a\epsilon)\,\tilde{e}_a = \\ &(\bar\epsilon_0^{+}\Gamma_4\epsilon_0^{-})\Big[\frac{ia\omega}{2}\cos\frac{\tau}{b}\tilde{e}_1 - \frac{ia\omega}{2}\frac{\rho}{b}\sin\frac{\tau}{b}\tilde{e}_0 + \Big(\frac{3i\omega ab}{2l}f\sin\frac{\tau}{b}+1\Big)\tilde{e}_4 + \Big(\frac{-3\omega ab}{2l}+if\sin\frac{\tau}{b}\Big)\tilde{e}_9\Big] \\ &+(\bar\epsilon_0^{-}\Gamma_4\epsilon_0^{+})\Big[\frac{-ia\omega}{2}\cos\frac{\tau}{b}\tilde{e}_1 + \frac{ia\omega}{2}\frac{\rho}{b}\sin\frac{\tau}{b}\tilde{e}_0 + \Big(\frac{-3i\omega ab}{2l}f\sin\frac{\tau}{b}+1\Big)\tilde{e}_4 + \Big(\frac{-3\omega ab}{2l}-if\sin\frac{\tau}{b}\Big)\tilde{e}_9\Big] \\ &+(\bar\epsilon_0^{+}\Gamma_4\epsilon_0^{+})\Big[-\frac{l}{3b}\sin\frac{\tau}{b}\tilde{e}_1 - \frac{l}{3b}\Big(\frac{\rho}{b}\cos\frac{\tau}{b}-f\Big)\tilde{e}_0 + \Big(f\cos\frac{\tau}{b}-\frac{\rho}{b}\Big)\tilde{e}_4 + \frac{2l}{3\omega ab}\Big(f\cos\frac{\tau}{b}-\frac{\rho}{b}\Big)\tilde{e}_9\Big] \\ &+(\bar\epsilon_0^{-}\Gamma_4\epsilon_0^{-})\Big[-\frac{l}{3b}\sin\frac{\tau}{b}\tilde{e}_1 - \frac{l}{3b}\Big(\frac{\rho}{b}\cos\frac{\tau}{b}+f\Big)\tilde{e}_0 + \Big(f\cos\frac{\tau}{b}+\frac{\rho}{b}\Big)\tilde{e}_4 + \frac{2l}{3\omega ab}\Big(f\cos\frac{\tau}{b}+\frac{\rho}{b}\Big)\tilde{e}_9\Big]\,. \end{aligned} \tag{5.24}$$

We have checked that these are Killing vectors of the near-horizon metric. Expressed in the holonomic basis these are:

$$\begin{aligned} v^{(1)} &= -\frac{ia\omega}{2bf}\rho\sin\frac{\tau}{b}\partial_\tau + \frac{ia\omega f}{2}\cos\frac{\tau}{b}\partial_\rho + \Big(\frac{2ab}{\omega} + \frac{3ia^2b^2}{lf}\sin\frac{\tau}{b}\Big)\partial_\phi + \Big(\frac{2ab\omega}{l^2} - \frac{ia^2b^2}{fl}\sin\frac{\tau}{b}\Big)\partial_{\underline{\xi}} \\ v^{(2)} &= v^{(1)\,*} \\ v^{(3)} &= \frac{l}{3bf}\Big(f - \frac{\rho}{b}\cos\frac{\tau}{b}\Big)\partial_\tau - \frac{fl}{3b}\sin\frac{\tau}{b}\partial_\rho + \frac{2ab}{\omega f}\cos\frac{\tau}{b}\partial_\phi - \frac{2ab}{3\omega f}\cos\frac{\tau}{b}\partial_{\underline{\xi}} \\ v^{(4)} &= -\frac{l}{3bf}\Big(f + \frac{\rho}{b}\cos\frac{\tau}{b}\Big)\partial_\tau - \frac{fl}{3b}\sin\frac{\tau}{b}\partial_\rho + \frac{2ab}{\omega f}\cos\frac{\tau}{b}\partial_\phi - \frac{2ab}{3\omega f}\cos\frac{\tau}{b}\partial_{\underline{\xi}}\,. \end{aligned} \tag{5.25}$$

The generators of the purely bosonic isometries do not change in the global coordinates.

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6. Poincaré D-brane probes

In this section we initiate the study of probe branes in the near-horizon geometry. To establish our conventions we quote here the D3-brane action we shall be working with:

$$S_{\mathrm{D3}} = -\left[\int_{\mathrm{D3}} \mathrm{dvol} \pm C^{(4)}\right] . \tag{6.1}$$

In this expression, dvol is the volume form associated to the induced metric on the world volume, which we denote by h, and $C^{(4)}$ is the pull back of the four-form potential. The positive sign is for a brane and the negative sign for an anti-brane. The conserved charges will be specified using the point particle Lagrangian denoted by L obtained after integrating over all the spatial coordinates. From a world-volume perspective, supersymmetry of a configuration can be established by studying the kappa-symmetry condition. We say that an (anti-) brane is supersymmetric if it obeys an equation of the form

$$\Gamma\epsilon = \pm i\epsilon . \tag{6.2}$$

The negative sign is for a brane and the positive sign for an anti-brane. The spinor ϵ is the background Killing spinor derived above. Here Γ is the kappa-projection matrix, defined as

$$\begin{aligned}\Gamma &= \frac{1}{4!}\frac{1}{\sqrt{-h}}\epsilon^{\sigma_i\sigma_j\sigma_k\sigma_l}\gamma_{\sigma_i\sigma_j\sigma_k\sigma_l} \\ &= -\frac{1}{\sqrt{-h}}(\gamma_0\gamma_1\gamma_2\gamma_3 + (-h_{01}\gamma_{23} - h_{03}\gamma_{12} + h_{02}\gamma_{13} + h_{13}\gamma_{02} - h_{12}\gamma_{03} - h_{23}\gamma_{01}) \\ &\quad +(h_{23}h_{01} + h_{12}h_{03} - h_{13}h_{20}))\,,\end{aligned} \tag{6.3}$$

and γ_{σ_i} are the world volume gamma matrices

$$\gamma_{\sigma_i} = \partial_{\sigma_i}X^\mu\Gamma_\mu . \tag{6.4}$$

and $\gamma_{\sigma_i\sigma_j} = \frac{1}{2}(\gamma_{\sigma_i}\gamma_{\sigma_j} - \gamma_{\sigma_j}\gamma_{\sigma_i})$. The world-volume gamma matrices satisfy $\{\gamma_{\sigma_i}, \gamma_{\sigma_j}\} = 2h_{\sigma_i\sigma_j}$. As in (6.3), we will sometimes find it convenient to use the shorthand $\gamma_i = \gamma_{\sigma_i}$ for world-volume indices.

6.1 Solving the equations of motion

In Poincaré coordinates one can write the 5-form RR field strength as $F^{(5)} = dC^{(4)}$ where

$$\begin{aligned}C^{(4)} &= \frac{2\omega}{\lambda}e^{0234} + \cot\alpha\, e^{678}\wedge\left(e^9 + \frac{2}{\sqrt{3}}A\right) \\ &\quad -\frac{2}{\sqrt{3}}\left[A\wedge(e^{57}+e^{68})\wedge\left(e^9+\frac{2}{\sqrt{3}}A\right) + \frac{l}{2}\left(e^9+\frac{2}{\sqrt{3}}A\right)\wedge\left(\star F + \frac{2}{\sqrt{3}}A\wedge F\right)\right] ,\end{aligned} \tag{6.5}$$

with

$$\star F + \frac{2}{\sqrt{3}}A\wedge F = \frac{\sqrt{3}}{l}\left[e^0\wedge(e^{23}-e^{14}) + \frac{l\omega^2}{4}\left(1+\frac{2\omega^2}{l^2}\right)\sigma_{123}\right] . \tag{6.6}$$

6.1.1 Giant probes

Let us now turn to probe D3-branes that wrap a sub-manifold of the deformed S^5 part of the geometry similar to the giant gravitons of pure AdS. We choose the following static-gauge ansatz

$$t = \sigma_0, \quad \beta = \sigma_1, \quad \xi_2 = \sigma_2, \quad \xi_3 = \sigma_3 \tag{6.7}$$

with the rest of the coordinates assumed to be functions of σ_0 only. The DBI part of the action follows from[2]

$$\begin{aligned}\sqrt{-\det h_{\sigma_i\sigma_j}} = \frac{l^2}{4\omega}\cos\sigma_1\sin\sigma_1\cos^3\alpha \Bigg[& \cos^2\alpha\left(\omega^3\Sigma_3 + 8lr\right)^2 - 64l\omega r\left(\omega^2\Sigma_3 - l^2\sin^2\alpha\dot{\xi}_1\right) \\ & -4\omega^2\left(\omega^2\Sigma_3^2(l^2+\omega^2) + \omega^2 l^2\left(\sin^2\theta\dot{\psi}^2 + \dot{\theta}^2 + 2\Sigma_3\sin^2\alpha\dot{\xi}_1\right)\right. \\ & \left.\left. +4l^4\left(\dot{\alpha}^2 + \sin^2\alpha\dot{\xi}_1^2\right)\right) - \frac{4l^4\omega^4\dot{r}^2}{\lambda^2 r^2}\right]^{1/2}, \end{aligned} \tag{6.8}$$

where $\Sigma_3 = \dot{\phi} + \cos\theta\dot{\psi}$. The WZ coupling for these configurations is

$$C^{(4)}_{\sigma_0\sigma_1\sigma_2\sigma_3} = l^3\left(l\dot{\xi}_1 + \frac{2r}{\omega} + \frac{\omega^2}{4l}\Sigma_3\right)\cos^4\alpha\sin\sigma_1\cos\sigma_1\,. \tag{6.9}$$

It can be verified that for $\dot{\xi}_1 = \dot{\theta} = \dot{\phi} = \dot{\psi} = \dot{\alpha} = \dot{r} = 0$ all equations of motion for an anti-brane are satisfied identically. These giant-like solutions carry non-zero angular momentum given by

$$P_\phi = 2\pi^2 l^2\omega^2\cos^2\alpha, \quad P_{\xi_1} = 2\pi^2 l^4\cos^2\alpha. \quad P_\psi = 2\pi^2 l^2\omega^2\cos^2\alpha\cos\theta\,. \tag{6.10}$$

The giant like solutions found here have $H = 0$. Note that $P_\phi|_{\max} = 2\pi^2 l^2\omega^2$, $P_{\xi_1}|_{\max} = 2\pi^2 l^4$, and $P_\psi|_{\max} = 2\pi^2 l^2\omega^2$ which suggest a stringy exclusion principle at work.

6.1.2 Dual-giant probes

Now we look for solutions that are analogous to the dual giant gravitons in AdS in that their world volume takes up an S^3 in the five-dimensional part of our geometry. Choosing static gauge, our ansatz is

$$t = \sigma_0, \quad \theta = \sigma_1, \quad \phi = \sigma_2, \quad \psi = \sigma_3 \tag{6.11}$$

with all other coordinates assumed to be functions of σ_0 only. Thus the DBI contribution to the action follows from

$$\sqrt{-\det h_{\sigma_i\sigma_j}} = \frac{\omega^{5/2}}{16\,l}\sin\sigma_1\left[\omega\left(8r + l\omega\sum_{i=1}^{3}\mu_i^2\,\dot{\xi}_i\right)^2 - 4l(l^2+\omega^2)\sum_{i=1}^{3}\mu_i^2\,\dot{\xi}_i(4r + l\omega\dot{\xi}_i)\right]^{1/2}. \tag{6.12}$$

[2] In this section we quote the full action for completeness. There are terms that can be dropped consistently from the action without changing the equations of motion for the class of solutions we are interested in. We will drop such terms from now on to avoid clutter.

Without loss of generality we have dropped terms involving $\dot{\alpha}, \dot{\beta}$ and $\dot{r}$ that do not contribute to the equations of motion for the configurations we are about to study. The pull back of the four-form potential is

$$C^{(4)}_{\sigma_0\sigma_1\sigma_2\sigma_3} = -\left[\frac{4r\omega^3}{l} + l^2\omega^2\left(1+\frac{\omega^2}{2l^2}\right)\sum_{i=1}^{3}\mu_i^2\dot{\xi}_i\right]\frac{1}{8}\sin\sigma_1 \tag{6.13}$$

To find solutions we first note that since the Lagrangian depends only on $\dot{\xi}_i$'s putting $\ddot{\xi}_i = 0$ would solve the ξ_i e.o.m . Setting $\dot{\xi}_i = 0$ solves the equations of motion and gives the Hamiltonian $H = -L$. We find for the momenta conjugate to the angular variables $P_{\xi_i} = \frac{\partial L}{\partial \dot{\xi}_i}$

$$P_{\xi_i} = 3\pi^2\omega^2 l^2\left(1+\frac{\omega^2}{3l^2}\right)\mu_i^2\,. \tag{6.14}$$

This means that $\sum_{i=1}^{3} P_{\xi_i} = 3\pi^2\omega^2 l^2\left(1+\frac{\omega^2}{3l^2}\right)$ on our solutions. Furthermore we find that $H = 0$.

If we use the coordinates $\tilde{t}$ and $\tilde{\phi}$, which give the asymptotic geometry of $AdS_5 \times S^5$ in the standard global coordinates, then we see that the vanishing Hamiltonian actually implies $E = -\frac{2}{l}J$ where J is the spin of the probe branes when measured in the new coordinates. When one considers multiple configurations of dual-giants in $AdS_5 \times S^5$ there is an upper limit on their number given by the number of units of flux through the 5-sphere [28, 29]. In our case too one expects that there is an upper limit on the number of dual-giants.

6.2 Supersymmetry

Let us now investigate the kappa symmetry conditions for the configurations introduced above.

6.2.1 Giant probes

For the solutions (6.7) we find the world-volume gamma matrices

$$\gamma_0 = \frac{4r}{\omega}\Gamma_0 P_+\,, \tag{6.15}$$

$$\gamma_1 = l\cos\alpha\Gamma_6\,, \tag{6.16}$$

$$\gamma_2 = -l\cos\alpha\sin\sigma_1(-\Gamma_8\cos\sigma_1 + (\Gamma_9\cos\alpha + \Gamma_7\sin\alpha)\sin\sigma_1)\,, \tag{6.17}$$

$$\gamma_3 = -l\cos\alpha\cos\sigma_1(\Gamma_8\sin\sigma_1 + (\Gamma_9\cos\alpha + \Gamma_7\sin\alpha)\cos\sigma_1)\,, \tag{6.18}$$

On the solution $\sqrt{-h} = \frac{1}{\omega}2l^3 r\cos^4\alpha\sin\beta\cos\beta$. Thus, using equation (6.3) we get

$$\Gamma = i\sec\alpha[-2\Gamma_0 P_+(\cos\alpha\Gamma_9 - \sin\alpha\Gamma_7) - \cos\alpha]\,. \tag{6.19}$$

And hence

$$\Gamma\epsilon = i\epsilon\,, \tag{6.20}$$

for $\epsilon = P_-\eta$, η being the Killing spinor in Poincaré coordinates with $P_+\eta = 0$. This sets $\epsilon_0^+ = 0$. Hence these configurations are half-BPS with respect to the near-horizon preserving precisely the supersymmetries of the full black hole. The isometry preserved by the brane can be determined by adopting a similar procedure as in section 4. The Killing vector preserved by the brane is proportional to ∂_t which is just the Hamiltonian. Equating this to zero gives us the $H = 0$ condition obtained from the equations of motion.

6.2.2 Dual-giant probes

For the solutions (6.11) we find the world-volume gamma matrices

$$\gamma_0 = \frac{4r}{\omega}\Gamma_0 P_+ \,, \tag{6.21}$$

$$\gamma_1 = \frac{\omega}{2}(\sin\sigma_2\Gamma_2 + \cos\sigma_2\Gamma_3) \,, \tag{6.22}$$

$$\gamma_2 = -\frac{\omega}{4l}(3\omega\Gamma_0 - 2\lambda\Gamma_4 + \omega\Gamma_9) \,, \tag{6.23}$$

$$\gamma_3 = \cos\sigma_1\gamma_2 - \frac{\omega}{2}\sin\sigma_1(\cos\sigma_2\Gamma_2 - \sin\sigma_2\Gamma_3) \,. \tag{6.24}$$

On the solution $\sqrt{-h} = \frac{\omega^3}{2l} r \sin\sigma_1$. Using (6.3), we calculate

$$\Gamma = \frac{i}{2\omega}\left[\Gamma_0 P_+(3\omega\Gamma_0 - 2\lambda\Gamma_4 + \omega\Gamma_9) + 2\omega\right] . \tag{6.25}$$

With this we find $\Gamma\epsilon = -i\epsilon$ for $\epsilon = P_-\eta$ with η the Killing spinor in Poincaré coordinates and $P_+\eta = 0$, as in the previous case. Hence these probes are half-BPS with respect to the near-horizon. As in the previous case, the Killing spinor bilinear implies $H = 0$, consistent with the equations of motion. Thus both the solutions preserve only the supersymmetries of the full black hole.

7. Global D-brane probes

In this section we exhibit some half-BPS D3-brane probes in the near horizon geometry in global coordinates.

7.1 Solving the equations of motion

In global coordinates we can take the 4-form RR potential to be

$$\begin{aligned} C^{(4)} &= \frac{4\rho}{lf}e^{0234} + \cot\alpha\, e^{678} \wedge \left(e^9 + \frac{2}{\sqrt{3}}A\right) \\ &\quad -\frac{2}{\sqrt{3}}\left[A\wedge(e^{57}+e^{68})\wedge\left(e^9+\frac{2}{\sqrt{3}}A\right) + \frac{l}{2}\left(e^9+\frac{2}{\sqrt{3}}A\right)\wedge\left(\star F + \frac{2}{\sqrt{3}}A\wedge F\right)\right], \\ \star F + \frac{2}{\sqrt{3}}A\wedge F &= \frac{\sqrt{3}}{2}\left[\frac{2ab}{\omega}\left(1+\frac{\omega^2}{2l^2}\right)e^{234} + \frac{2}{l}e^{014} + \frac{2ab\rho}{\omega l f}e^{023}\right]. \end{aligned} \tag{7.1}$$

7.1.1 Giant probes

We now exhibit a two classes of solutions to the DBI action of the D3-brane probes in global coordinates. We first choose

$$\tau = \sigma_0, \quad \beta = \sigma_1, \quad \xi_2 = \sigma_2, \quad \xi_3 = \sigma_3 \tag{7.2}$$

with the rest of the coordinates functions of σ_0. The DBI contribution to the action follows from

$$\sqrt{-\det h_{\sigma_i\sigma_j}} = \frac{1}{8\omega} l^2 \cos^3\alpha \sin 2\beta \Bigg[\frac{64\rho^2 l^2}{a^2 b^2} \cos^2\alpha + \frac{8 l\omega\rho}{ab} \left(-8l^2 \sin^2\alpha \dot{\xi}_1 + 2\omega^2(\cos^2\alpha - 4) \right) \\ + \omega^2 \Big(16 l^2 - 16 l^4 \sin^2\alpha \dot{\xi}_1^2 + 8 l^2 \omega^2 \sin^2\alpha \dot{\xi}_1 \dot{\phi} \\ + \omega^2 \dot{\phi}^2 (-8l^2 - 4\omega^2 + \omega^2 \cos^2\alpha) \Big) \Bigg]^{1/2} . \tag{7.3}$$

The WZ coupling is

$$C^{(4)}_{\sigma_0\sigma_1\sigma_2\sigma_3} = l^4 \cos^4\alpha \cos\sigma_1 \sin\sigma_1 \left[\dot{\xi}_1 - \frac{\omega^2}{4l^2} \Sigma_3 + \frac{2\rho}{\omega l a b} \right] , \tag{7.4}$$

where, as before, $\Sigma_3 = \dot{\phi} + \cos\theta\dot{\psi}$ and without loss of generality we have dropped terms involving $\dot{\rho}, \dot{\alpha}, \dot{\theta}$ and $\dot{\psi}$ which do not contribute to the equations of motion. One can verify that

$$|\dot{\phi}| = \frac{2l}{\omega\lambda}, \quad |\dot{\xi}_1| = \frac{2\omega}{l\lambda}, \quad \dot{\psi} = 0 \tag{7.5}$$

are solutions to the Lagrangian $\mathcal{L} = -\sqrt{-\det h_{\sigma_i\sigma_j}} \pm C^{(4)}_{\sigma_0\sigma_1\sigma_2\sigma_3}$ for any constant value of α, ψ and θ, provided $\rho > -\rho_g$ for branes and $\rho < \rho_g$ for anti-branes where

$$\rho_g = \frac{3\omega a b^2}{2l} .$$

One must take $\dot{\phi}$ and $\dot{\xi}_1$ positive for an anti-brane and negative for a brane. The conserved charges for these solutions are

$$P_\phi = \left(\frac{2\pi^2}{3} l^4 + \frac{4\pi^2 a\rho\omega^2 l^3}{\rho \pm \rho_g} \right) \cos^2\alpha, \quad P_{\xi_1} = 2\pi^2 l^4 \cos^2\alpha , \tag{7.6}$$

with the above sign for branes and below for anti-branes. Note that P_ϕ is infinite at $\rho = \rho_g$ while P_{ξ_1} is independent of ρ. We will demonstrate later on that supersymmetry dictates $\rho = 0$. For this, the maximum value of the momenta are $P_\phi|_{\max} = \frac{2\pi^2 l^4}{3}$, $P_{\xi_1}|_{\max} = 2\pi^2 l^4$ again suggesting a stringy exclusion principle at work. It is easy to verify that the Lagrangian vanishes and the Hamiltonian is given by

$$H = \frac{2l}{\omega\lambda} |P_\phi| + \frac{2\omega}{l\lambda} |P_{\xi_1}| \tag{7.7}$$

This is actually the relation expected for BPS objects. To see this one can verify that the following Killing vector of the background

$$\frac{2\lambda}{3\omega} \partial_\tau + \frac{4l}{3\omega^2} \partial_\phi + \frac{4}{3l} \partial_{\xi_1} \tag{7.8}$$

is preserved by the probe brane solutions above for $\rho = 0$. This can be seen by considering the bilinears of the supersymmetries preserved by the probe branes similar to those in section 4 and 5. Then identifying the generators ∂_τ, ∂_ϕ and ∂_{ξ_1} with the charges H, P_ϕ and P_{ξ_1} respectively in eq. (7.8) and equating it to zero results in the BPS equation.

There is another class of solutions which have $\dot\psi \neq 0$ as well. It is easy to verify that

$$\theta = 0, \quad \dot\phi = \dot\psi = \frac{\eta\, l}{\omega\lambda}, \quad \dot\xi_1 = \frac{2\eta\omega}{l\lambda} \tag{7.9}$$

and

$$\theta = \pi, \quad \dot\phi = -\dot\psi = \frac{\eta l}{\omega\lambda}, \quad \dot\xi_1 = \frac{2\eta\omega}{l\lambda} \tag{7.10}$$

are solutions to the action $\mathcal{L} = -\sqrt{-\det h_{\sigma_i\sigma_j}} + \eta\, C^{(4)}$ for $\eta = \pm 1$ whenever

$$\rho \leq \rho_g\,. \tag{7.11}$$

The solutions at $\theta = 0$ have

$$P_{\xi_1} = \eta\, 4\pi^2 l^4 \cos^2\alpha, \quad P_\phi = P_\psi = \eta\, \frac{2\pi^2 l^2 (l^2 + 3\omega^2 \rho/\rho_g)}{3(1-\rho/\rho_g)} \cos^2\alpha, \tag{7.12}$$

and those at $\theta = \pi$ have

$$P_{\xi_1} = \eta\, 4\pi^2 l^4 \cos^2\alpha, \quad P_\phi = -P_\psi = \eta\, \frac{2\pi^2 l^2 (l^2 + 3\omega^2 \rho/\rho_g)}{3(1-\rho/\rho_g)} \cos^2\alpha. \tag{7.13}$$

These configurations have vanishing Lagrangian and therefore their Hamiltonian is[3]

$$H = \frac{l}{\omega\lambda}(|P_\phi| + |P_\psi|) + \frac{2\omega}{l\lambda}|P_{\xi_1}| = \frac{2l}{\omega\lambda}|P_\phi| + \frac{2\omega}{l\lambda}|P_{\xi_1}|. \tag{7.14}$$

7.1.2 Dual-giant probes

Let us assume the most general ansatz (in static gauge) for a dual-giant graviton in global coordinates:

$$\tau = \sigma_0, \quad \theta = \sigma_1, \quad \phi = \sigma_2, \quad \psi = \sigma_3, \tag{7.15}$$

where all other embedding coordinates are functions of σ_0. The DBI contribution to the action may be written in the form

$$\begin{aligned}\sqrt{-\det h_{\sigma_i\sigma_j}} = \frac{\omega^{5/2}}{16 l} \sin\sigma_1 \Bigg(&\omega \left[\frac{8\rho}{ab} + l\omega \sum_{i=1}^3 \mu_i^2 \dot\xi_i\right]^2 \\ &-4(l^2+\omega^2)\left[\omega\left(-1 + l^2 \sum_{i=1}^3 \mu_i^2 \dot\xi_i^2\right) + \frac{4l\rho}{ab}\sum_{i=1}^3 \mu_i^2\dot\xi_i\right]\Bigg)^{1/2}.\end{aligned} \tag{7.16}$$

where $h_{\sigma_i\sigma_j}$ denotes the induced metric on the world volume of the dual giant. Without loss of generality we have dropped terms involving $\dot\alpha$, $\dot\beta$ and $\dot\rho$ that do not affect the equations of motion on the configurations we are about to study. The induced four-form is

$$C^{(4)}_{\sigma_0\sigma_1\sigma_2\sigma_3} = -\frac{\omega^3}{8}\left[\frac{1}{l\lambda}(l^2+\omega^2) + \frac{l}{\omega}\left(1 + \frac{\omega^2}{2l^2}\right)\sum_{i=1}^3 \mu_i^2 l\, \dot\xi_i + \frac{4\rho}{lab}\right]\sin\sigma_1\,, \tag{7.17}$$

[3] One can express the Hamiltonian in manifestly gauge-invariant form in terms of the gauge-invariant momenta $\Pi_i = P_i - A_i$, where A_i is the effective particle gauge potential. To do so, one should fix the constant appearing in the WZ term by demanding $L - \dot X_i A_i = 0$. This effectively removes the piece coming from the WZ term. In this case $H = \frac{2l}{\omega\lambda}|\Pi_\phi| + \frac{2\omega}{l\lambda}|\Pi_{\xi_1}|$.

where we have chosen to add a constant for convenience which does not change the equations of motion. Then one can verify that

$$\dot{\xi}_1 = \dot{\xi}_1 = \dot{\xi}_3 = \frac{2\omega}{l\lambda} \tag{7.18}$$

the equations of motion of the action $\mathcal{L} = -\sqrt{-\det h_{\sigma_i\sigma_j}} + C^{(4)}$ are satisfied when

$$\rho \leq \rho_{dg} := \frac{lab^2}{2\omega}\,. \tag{7.19}$$

These solutions have the following conserved charges

$$P_{\xi_i} = \pi^2\omega^2\left[\frac{3\omega^4 + 4l^2(\omega^2 + \omega\rho l/(2ab^2))}{l^2 - 4\omega\rho l/(2ab^2)} - (2l^2 + \omega^2)\right]\mu_i^2 \tag{7.20}$$

for $i = 1$, 2 and 3. Notice that these angular momenta diverge as the radial positions of the dual-giants ρ approaches ρ_{dg}. Furthermore this critical value of the radial coordinate is different from what the giants see which is $3\omega^2/l^2$ times ρ_{dg}. The Lagrangian evaluated on the configurations again vanishes and so the Hamiltonian[4] is given by

$$H = \frac{2\omega}{l\lambda}(|P_{\xi_1}| + |P_{\xi_2}| + |P_{\xi_3}|) \tag{7.21}$$

which also diverges at $\rho = \rho_{dg}$. The same analysis can be repeated for branes with $\rho_{dg} \to -\rho_{dg}$ and changing the signs of $\dot{\xi}_i$'s.

7.2 Supersymmetry

In this section we analyse the amount of supersymmetry preserved by the probes in global coordinates.

7.2.1 Giant probes

First consider $\dot{\psi} = 0$. The pull-back gamma matrices are given by

$$\begin{aligned}
\gamma_0 &= f\Gamma_0 - \left(\frac{3\rho}{l} \mp \frac{2}{a\omega}\right)\Gamma_4 + \left(\pm\frac{b}{l} - \frac{2\rho}{\omega ab}\right)\Gamma_9 \pm \frac{4b}{l}\sin\alpha(\Gamma_7\cos\alpha - \Gamma_9\sin\alpha)\,, && (7.22)\\
\gamma_1 &= l\cos\alpha\Gamma_6\,, && (7.23)\\
\gamma_2 &= -l\cos\alpha\sin\sigma_1\left(-\cos\sigma_1\Gamma_8 + \sin\sigma_1(\cos\alpha\Gamma_9 + \sin\alpha\Gamma_7)\right)\,, && (7.24)\\
\gamma_3 &= -l\cos\alpha\cos\sigma_1\left(\sin\sigma_1\Gamma_8 + \cos\sigma_1(\Gamma_9\cos\alpha + \Gamma_7\sin\alpha)\right)\,, && (7.25)
\end{aligned}$$

where the upper sign is for an anti-brane and the lower sign for a brane. Using these we get

$$\Gamma = \frac{il^3\cos^3\alpha\sin\sigma_1\cos\sigma_1}{\sqrt{-h}}\left(\pm\frac{6b}{l} + \frac{4\rho}{\omega ab}\right)\left(\frac{1}{2}\cos\alpha + \sin\alpha\Gamma_{79}P_+^{1,2} - \cos\alpha P_+^{1,2}\right)\,, \tag{7.26}$$

[4]The gauge-invariant form of the Hamiltonian as described in footnote 3 is given by $H = \frac{2\omega}{l\lambda}(|\Pi_{\xi_1}| + |\Pi_{\xi_2}| + |\Pi_{\xi_3}|)$.

where

$$P_+^{1,2} = \frac{1}{2}\left[1 + \left(\pm\frac{3b}{l} + \frac{2\rho}{\omega ab}\right)^{-1}\left(f\,\Gamma_{09} + \left(-\frac{3\rho}{l} \pm \frac{2}{a\omega}\right)\Gamma_{49}\right)\right],$$

and can be shown to be a projector. We further define the orthogonal projectors to be

$$P_-^{1,2} = \frac{1}{2}\left[1 - \left(\pm\frac{3b}{l} + \frac{2\rho}{\omega ab}\right)^{-1}\left(f\Gamma_{09} + \left(-\frac{3\rho}{l} \pm \frac{2}{a\omega}\right)\Gamma_{49}\right)\right].$$

Hence, if we choose $\epsilon = P_-^{1,2}\eta$ with $P_+^{1,2}\eta = 0$, η being the Killing spinor in global coordinates, then it is easy to see that $\Gamma\epsilon = \pm i\epsilon$ and that the configurations are thus half-BPS with respect to the near-horizon. We also see that the projectors are ill defined at $\rho = \pm\rho_g$ where the upper sign is for a brane and lower for anti-brane. These are the same positions where the equations are not solvable for the corresponding cases. We must further ensure that $P_+^{1,2}\eta = 0$. First consider an anti-brane. Write

$$P_+^1 = \frac{1}{2}(1 + A\Gamma_{09} + B\Gamma_{49})$$

and the Killing spinor as

$$\begin{aligned}\eta &= \left(e^{-\frac{\chi}{2}}\cos\frac{\tau}{2b} + ie^{\frac{\chi}{2}}\sin\frac{\tau}{2b}\Gamma_{49}\right)\epsilon_0^+ + \left(e^{\frac{\chi}{2}}\cos\frac{\tau}{2b} - ie^{-\frac{\chi}{2}}\sin\frac{\tau}{2b}\Gamma_{49}\right)\epsilon_0^- \\ &= (f_+ + ig_+\Gamma_{49})\epsilon_0^+ + (f_- + ig_-\Gamma_{49})\epsilon_0^-,\end{aligned} \tag{7.27}$$

with $\rho = b\sinh\chi$. Some useful relations are

$$\Gamma_{09}\epsilon_0^\pm = \pm\frac{2b}{l}\left(-\frac{3}{2}\Gamma_{49} + \frac{l}{\omega ab}\right)\epsilon_0^\pm, \tag{7.28}$$

$$\Gamma_{04}\epsilon_0^\pm = \pm\frac{2b}{l}\left(\frac{3}{2} + \frac{l}{\omega ab}\Gamma_{49}\right)\epsilon_0^\pm. \tag{7.29}$$

We demand $P_+^1\eta = 0$ corresponding to $P_-^1\eta$ being preserved. Now we note that $\epsilon_0^\pm$ and $\Gamma_{49}\epsilon_0^\mp$ have the same chirality. This leads to the following equations on equating the coefficient of $\cos\frac{\tau}{2b}$

$$e^{-\chi/2}\left(1 + \frac{2A}{\omega a}\right)\epsilon_0^+ = -e^{\chi/2}\left(\frac{3bA}{l} + B\right)\Gamma_{49}\epsilon_0^-, \tag{7.30}$$

$$e^{\chi/2}\left(\frac{3bA}{l} + B\right)\epsilon_0^+ = -e^{-\chi/2}\left(1 + \frac{2A}{\omega a}\right)\Gamma_{49}\epsilon_0^-. \tag{7.31}$$

These lead to the conclusion that $\rho = 0$ and $\epsilon_0^+ = -\Gamma_{49}\epsilon_0^-$. It can be verified that these conditions satisfy the equations obtained from the coefficients of $\sin\frac{\tau}{2b}$ as well. The same calculation can be repeated for the brane case. The conclusion is that the condition on the constant spinors is $\epsilon_0^+ = \pm\Gamma_{49}\epsilon_0^-$, the upper sign for brane and lower for anti-brane and $\rho = 0$.

The calculation for non-zero $\dot\psi$ can be repeated in a similar manner. It turns out that the world-volume gamma matrices are identical to the above case and hence the supersymmetry analysis is identical to the one given there.

7.2.2 Dual giants

The world-volume gamma matrices are

$$\gamma_0 = -\frac{2}{l}\rho\left(\frac{3}{2}\Gamma_4 + \frac{l}{\omega ab}\Gamma_9\right) + f\ \Gamma_0 \pm \frac{4b}{l}\Gamma_9 \tag{7.32}$$

$$\gamma_1 = \frac{\omega}{2}\left(\cos\phi\Gamma_3 + \sin\phi\Gamma_2\right) \tag{7.33}$$

$$\gamma_2 = \frac{\omega}{2l}\left(\frac{\omega}{2}\Gamma_9 + \frac{l}{ab}\Gamma_4\right) \tag{7.34}$$

$$\gamma_3 = \cos\theta\gamma_2 + \frac{\omega}{2}\sin\theta\left(\sin\phi\Gamma_3 - \cos\phi\Gamma_2\right)\ , \tag{7.35}$$

where the upper sign is for brane and lower for anti-brane. After some algebra one can derive the following simple expression for the Kappa-symmetry projection matrix

$$\Gamma = \frac{i}{\sqrt{-h}}\left(\frac{\omega}{2}\right)^2 \sin\theta(h_{02} - \gamma_0\gamma_2)\,, \tag{7.36}$$

with

$$h_{02} - \gamma_0\gamma_2 = \frac{\omega}{2l}\left[\left(\frac{2l}{\omega}\rho \mp \frac{4}{a}\right)\Gamma_{49} + f\ \left(\frac{\omega}{2}\Gamma_{09} + \frac{l}{ab}\Gamma_{04}\right)\right] \tag{7.37}$$

We note that

$$-\det h\mathbf{1} = \left(\frac{\omega}{2}\right)^4 \sin^2\theta\,(h_{02}\mathbf{1} - \gamma_0\gamma_2)^2 = \frac{\omega^6\sin^2\sigma_1}{4l^2a^2b^2}(\rho \pm \rho_{dg})^2\mathbf{1}\,, \tag{7.38}$$

the upper sign is for brane and lower for anti-brane. We can thus form the projectors $\mathcal{P}_\pm = \frac{1}{2}(1 \pm i\Gamma)$. From the above we see that $\mathcal{P}_\pm$ commutes with Γ^{0149}, Γ^{23} and Γ^{57}. Furthermore, the projectors become ill defined at $\rho = \rho_{dg}$ which is the same point where the angular momenta blow up. The condition on the constant spinors are derived as follows: For branes, we want to preserve $\Gamma\epsilon = -i\epsilon$. Let us write $\Gamma = c(A\Gamma_{49} + B\Gamma_{09} + C\Gamma_{04})$ where $c = \frac{i\omega^3}{8l\sqrt{-h}}\sin\sigma_1$. Then after some tedious algebra we get

$$\begin{aligned}\frac{\Gamma}{c}\epsilon = &\left(f_+\left(A - \frac{3bB}{l} + \frac{2C}{\omega a}\right) - ig_+\left(\frac{2B}{\omega a} + \frac{3bC}{l}\right)\right)\Gamma_{49}\epsilon_0^+ \\ &+ \left(-f_-\left(\frac{2B}{\omega a} + \frac{3bC}{l}\right) - ig_-\left(A + \frac{3Bb}{l} + \frac{2C}{\omega a}\right)\right)\epsilon_0^- \\ &+ \left(f_+\left(2\frac{B}{\omega a} + \frac{3bC}{l}\right) - ig_+\left(A + \frac{3Bb}{l} - \frac{2C}{\omega a}\right)\right)\epsilon_0^+ \\ &+ \left(f_-\left(A + \frac{3bB}{l} - \frac{2C}{\omega a}\right) + ig_-\left(\frac{2B}{\omega a} + \frac{3bC}{l}\right)\right)\Gamma_{49}\epsilon_0^-\,. \end{aligned} \tag{7.39}$$

Now we equate this to $-\frac{i}{c}\epsilon$. Equating the $\cos\frac{\tau}{2b}$ piece

$$f_+\left(A - \frac{3bB}{l} + \frac{2C}{\omega a}\right)\Gamma_{49}\epsilon_0^+ = f_-\left(\frac{2B}{\omega a} + \frac{3bC}{l} - \frac{i}{c}\right)\epsilon_0^-\,, \tag{7.40}$$

$$f_+\left(\frac{2B}{\omega a} + \frac{3bC}{l} + \frac{i}{c}\right)\epsilon_0^+ = -f_-\left(A + \frac{3bB}{l} - \frac{2C}{\omega a}\right)\Gamma_{49}\epsilon_0^-\,. \tag{7.41}$$

We can read off A, B, C from equation (7.37). This tells us that $\rho = 0$ and $\epsilon_0^+ = \Gamma_{49}\epsilon_0^-$. One can check that the other conditions arising from $\sin\frac{\tau}{2b}$ are also satisfied. Thus we conclude that, as for the giant case, supersymmetric dual giants also satisfy $\rho = 0$ and $\epsilon_0^+ = \pm\Gamma_{49}\epsilon_0^-$, the upper sign for branes and lower for anti-branes.

Conserved Killing vector

The calculation of the Killing vector that the giant and dual giants preserve is now straightforward. Imposing $\epsilon_0^+ = \pm\Gamma_{49}\epsilon_0^-$, we get the Killing spinor to simplify to

$$\epsilon = e^{-\frac{i\tau}{2b}}(1 \pm \Gamma_{49})\epsilon_0^+ \,. \tag{7.42}$$

Using this we find

$$\bar{\epsilon}\Gamma^a\epsilon\tilde{e}_a = 2\bar{\epsilon}_0^+ \left(-\Gamma_0\tilde{e}_0 \mp \Gamma_9\tilde{e}_4 \pm \Gamma_4\tilde{e}_9\right)\epsilon_0 \,. \tag{7.43}$$

Now using (5.23), the Killing vector becomes

$$v = -\frac{2\lambda}{3\omega}\tilde{e}_0 \mp \frac{2l}{3\omega ab}\tilde{e}_4 \pm \tilde{e}_9 \,, \tag{7.44}$$

which leads to

$$H = \frac{2\omega}{l\lambda}|P_{\underline{\xi}}| + \frac{2l}{\omega\lambda}|P_\phi| \,. \tag{7.45}$$

This is the expected BPS relation for both giants and dual giants.

8. Conclusion

In this paper, we considered the near-horizon geometry of the simplest of the supersymmetric AdS_5 black holes with two equal angular momenta and a single U(1) electric charge. It was shown that the isometry supergroup of the IIB uplift of this black hole is $\mathrm{SU}(1,1|1) \times \mathrm{SU}(2) \times U(3)$. This was achieved by explicitly constructing the Killing spinors of the geometry and then considering the bilinears following [17]. The near-horizon geometry has a deformed 3-sphere $\tilde{S}^3$ and a deformed 5-sphere $\tilde{S}^5$ with a fibration of the time coordinate of AdS_2 over them. We considered both Poincaré and global-like coordinates for the AdS_2 part of the geometry. We found that the number of supersymmetries of the near-horizon geometry of these black holes is twice that of the full solution.

We then exhibited several D3-brane configurations in this geometry that are analogous to the giant and dual-giant gravitons of the $AdS_5 \times S^5$ background. The dual-giant like D3-branes wrap the deformed-S^3 and the giant like objects wrap an S^3 inside the deformed-S^5 part of the geometry. In the Poincaré coordinates the branes do not rotate. They still carry non-zero angular momenta. In global coordinates the branes have to rotate in order to satisfy the equations of motion. All the configurations considered in Poincaré coordinates preserve two of the four supersymmetries. These two supersymmetries are simply those of the full black hole solution restricted to the near-horizon geometry.

We showed that the probes in global coordinates preserve two of the four supersymmetries of the background when placed at the centre $\rho = 0$ of AdS_2 and so are half-BPS. However, the configurations at a generic non-zero ρ do not preserve any supersymmetries.

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The D3-brane probes at generic ρ exhibit interesting features. In particular, they all satisfy a BPS-like energy condition and see a critical value of the radial position where their angular momenta diverge. It will be interesting to understand the physics behind this behaviour. We expect there to be more giant-type probe branes like those in [36]. There is a duality between configurations of giants and dual-giants in $AdS_5 \times S^5$. It will be interesting to see if such a duality holds in this case as well.

The results of this paper should help in counting microstates of the black hole under consideration as mentioned in the introduction. To make further progress in this direction one has to classify all the BPS objects in the global coordinates with a given set of supersymmetries. Then one should be able to quantise them using methods similar to [13, 14] (see also [32 − 35]) and count the different configurations with fixed quantum numbers.

There are several generalisations of the black holes considered here [3, 4, 24] which have non-equal angular momenta in AdS_5 directions and non-equal R-charges in S^5 directions (with one condition among them). However, we suspect that their near-horizon geometries again preserve four supersymmetries. The reason is that the generators of the bosonic part of the isometry group which are responsible for the generalisation do not participate in the supersymmetric part of the full supergroup of isometries. We expect that the near-horizons of the generalisations have the same supergroup part $\mathrm{SU}(1,1|1)$ of the isometries but with the bosonic parts $\mathrm{SU}(2)$ and $\mathrm{U}(3)$ broken to some subgroups of them. It will be interesting to establish this in detail.

Following Strominger et al [8, 9] one can ask what is the holographically dual conformal quantum mechanics of the string theory in the near-horizon geometry of the Gutowski-Reall black holes considered here. Our superisometries should be an important input in constructing the Lagrangian for such a quantum mechanics. One also expects that there are some small black holes with more supersymmetries than the Gutowski-Reall black holes (see [28] for instance). Counting the microstates in the near-horizon geometry of the Gutowski-Reall black holes might capture the entropies of the small black holes as well as in [37] in an analogous context. We hope to return to some of these questions in the future.

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